Partial-Wave Analysis of the Inelastic Scattering of Electrons by Nuclei. II. Application to the Liquid Drop Model*

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A partial-wave analysis of the solution of the Dirac equation in a Coulomb field is used to calculate inelastic scattering from a nucleus treated as an inhomogeneous charged liquid drop. Using charge density functions obtained from elastic electron scattering, the form factors for quadrupole excitation of the nuclei $Ni^{58,60}$ and Sr^{88} are calculated and found to display better agreement with experimental results than the usual Born approximation. To show the expected effects of a large Coulomb field, a similar calculation for Bi^{209} is also included.

INTRODUCTION

THE problem of Coulomb excitation of a nucleus by inelastic electron scattering has been discussed in an earlier paper[1] by the present authors. The method employed—partial-wave analysis of the Dirac wave function for the scattered electron—enables the effect of the Coulomb field to be taken into account for any scatterer which can be described in terms of a static charge distribution $\rho_0(r)$ and a transition charge distribution $\rho_{\text{trans}}(\mathbf{r},t)$, the small effect due to the nuclear current being neglected. It is interesting to consider the results of applying this method to a simple nuclear model and in the present work we have chosen the inhomogeneous liquid drop model used by Tassie.[2] This choice enables use to be made of the relatively well-determined parameters of the ground-state charge distribution in predicting the shape of the inelastic cross section for any multipole excitation. Another treatment using a quantized liquid drop has been given by Walecka.[3]

THE INHOMOGENEOUS LIQUID DROP MODEL

For details of this model the reader is referred to reference 2. The nuclear ground state is treated as a charged spherical liquid drop whose density $\rho_0(r)$ is a smooth function of distance from the center. Excitations of the drop are then described by a density function:

$$\rho(\mathbf{r},t)=\rho_0(r)+\rho_{\text{trans}}(\mathbf{r},t), \tag{1}$$

where, if it is further assumed that the liquid motion is irrotational and incompressible, the most general form of the transition charge density for an excitation of energy $\hbar\omega$ is

$$\rho_{\text{trans}}(\mathbf{r},t)=e^{i\omega t}\sum_{l=2}^{\infty}\sum_{m=-l}^{l} a_{lm}Y_l^m(\theta,\varphi)r^{l-1}(d/dr)\rho_0(r). \tag{2}$$

The shape of the radial part of the transition charge distribution for a 2^l-pole excitation [see Eq. (2.9) of reference 1] is then simply related to the static charge distribution:

$$\rho(r)\propto r^{l-1}(d/dr)\rho_0(r). \tag{3}$$

The choice of $\rho_0(r)$ is governed by the analysis of elastic scattering data for which a satisfactory fit has been obtained by Hofstadter *et al.*[4] using a charge distribution of the Fermi shape:

$$\rho_0(r)=\frac{1}{1+\exp[(r-a)/z_1]}. \tag{4}$$

Using the parameters given for Ni^{58}, Sr^{88}, and Bi^{209} (see Table I) the methods of reference 1 have been applied to calculate the form factors for quadrupole excitations in these nuclei. The form factor used for Ni^{60} is the same as that for Ni^{58}. There is indication from elastic scattering experiments[5] that the radii of the charge distributions of these isotopes differ by about 1%; but this does not change the shape of the form factor appreciably in the region of interest. Note that apart from normalization there are no arbitrary parameters in this calculation.

COMPARISON WITH EXPERIMENT

Helm[6] has measured the inelastic scattering of electrons from Sr^{88} at 187 MeV and gives experimentally determined form factors for low-lying quadrupole excitation. Similar results for $Ni^{58,60}$ and Co^{59} have been obtained by Crannell *et al.*[7] at 183 MeV.[8] Comparison of

TABLE I. Parameters used in the Fermi distribution as reported by Hofstadter.[a]

Nucleus	a (10^{-13} cm)	z_1 (10^{-13} cm)
$_{28}Ni^{58}$	4.28	0.566
$_{38}Sr^{88}$	4.80	0.523
$_{83}Bi^{209}$	6.25	0.614

[a] See reference 3.

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[8] A useful survey of observed transitions in inelastic electron scattering experiments is given by W. C. Barber, Ann. Rev. Nucl. Sci. (to be published). The nuclei we have chosen are those with mass number $A>50$ for which published data on $E2$ transitions are available at the time of writing. We would like to thank Dr. Barber for making a preprint of his paper available to us.

the results of these experiments with the present calculation is shown in Figs. 1, 2, and 3. Good agreement is obtained for Ni^{60} and Sr^{88} in the region of small momentum transfer corresponding to angles less than 70°

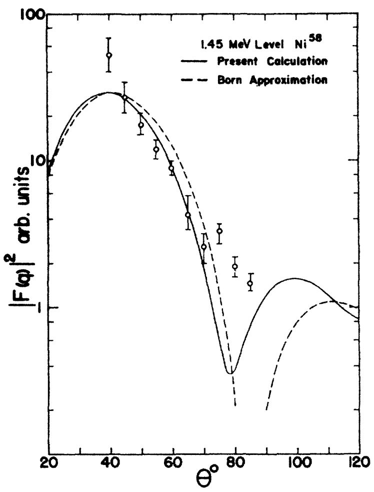

FIG. 1. The form factor for inelastic electron scattering producing quadrupole excitation in Ni^{58}. The experimental points are due to Crannell *et al.*[5]

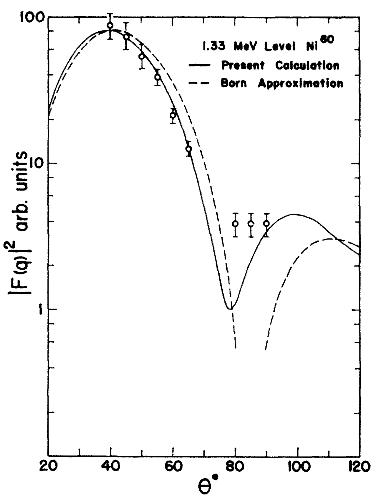

FIG. 2. The form factor for inelastic electron scattering producing quadrupole excitation in Ni^{60}. The experimental points are due to Crannell *et al.*[5]

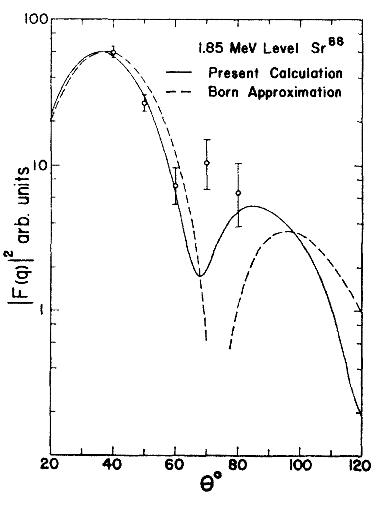

FIG. 3. The form factor for inelastic electron scattering producing quadrupole excitation in Sr^{88}. The experimental points are due to Helm.[4]

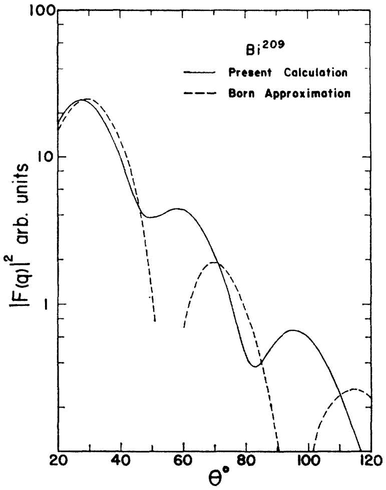

FIG. 4. The form factor for inelastic electron scattering producing quadrupole excitation in Bi^{209}.

at this energy. The discrepancy in the case of Ni^{58} is surprising. In view of the closed proton shell structure of nickel, little difference between the transition charge distributions of the isotopes Ni^{58} and Ni^{60} would be expected. In all cases the theoretical curves have the same character as the Born approximation fits obtained by Helm[6] and Crannell *et al.*[7] using an adjustable transition charge distribution. As expected, the Coulomb corrections replace the Born approximation zero with a minimum; however, the discrepancy with the experimental points persists in this region. To indicate the possible effects of a large Coulomb field we also include the results of calculation of a quadrupole excitation in bismuth [Fig. (4)].

CONCLUSIONS

The inclusion of Coulomb effects brings the theoretical prediction into considerably better agreement with experimental points in the region of low momentum transfer. The calculation predicts a well-defined minimum in the form factor for nuclei with $Z \lesssim 40$ which is not present in experimental results. This minimum is characteristic of any transition charge density confined to the region of the nuclear surface. It is, therefore, felt that there is definite physical information to be extracted from better data in this region which would justify more detailed fitting.

Similar calculations using positrons rather than electrons as the projectile are in progress. An extension of the calculations to $E3$ and $E4$ excitations is in course of preparation.

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